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Simultaneous axial conduction in the fluid and the pipe wall for forced convective laminar flow with blowing and suction at the wall

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Abstract—Numerical solutions are reported for conjugate heat transfer in a porous pipe having an internal flow with blowing or suction at the inner surface of the pipe and constant heat flux at the outer surface. The effect of the simultaneous axial conduction through the wall and the fluid has been studied for the combined hydrodynamic and thermal entry lengths. The results show that the ratio of the thermal conductivities of the pipe wall to the fluid and the thickness of the pipe wall may become significant factors on the heat transfer when the Peclet number is small, especially for the case when fluid is injected into the pipe. It is also shown that the effect of axial wall conduction for the case of constant heat flux at the outer wall surface can be neglected when the wall thickness is small and the ratio of the conductivities of the wall to the fluid approaches unity.

INTRODUCTION

CONJUGATE heat transfer in a pipe with internal laminar fluid flow is a problem of considering the simultaneous heat transfer inside the fluid and the solid wall. In conventional convective heat transfer problems, the thermal boundary condition at the solid–fluid interface is assumed to be known, either in terms of the heat flux or the temperature. Some considerations have been given in the past [1–11] concerning the errors that may be introduced by this assumption.

The primary research on the conjugate heat transfer problem for the circular tube was carried out by Luikov *et al.* [1], but their closed-form solution involved highly complicated functions. Because of this fact, no numerical results were reported. A detailed solution was given by Mori *et al.* [2, 3] for constant heat flux and constant temperature at the outer surface of the pipe. The solution was obtained by assuming that the velocity profile was the fully developed parabolic profile and by neglecting the axial conduction of the fluid. Mori *et al.* [3] also assumed that the wall–fluid interfacial temperature distribution in the axial direction was in the form of a power series with unknown coefficients. With this temperature distribution as the interfacial boundary condition, the analytical solution to the energy equation for the fluid was obtained using the Graetz solutions. The solution to the energy equation for the wall was then derived readily since all the boundary conditions were now known. Barozzi and Pagliarini [4] employed a finite element method in conjunction with Duhamel's theorem to extend the results given by Mori *et al.* [3] for different pipe lengths. Results in refs. [2–4] are applicable to cases with short heating sections due to the insulated boundary conditions imposed on the wall upstream and downstream of the heated section.

The coupled effect of axial conduction in the wall and the fluid in laminar pipe flow with an applied wall heat flux in the downstream region was studied for very long ducts numerically by Faghri and Sparrow [5] and Zariffeh *et al.* [6] and analytically by Campo and Rangel [7] and Soliman [8]. In these analyses the pipe is extended indefinitely upstream and downstream of the heating section. Faghri and Sparrow [5] investigated simultaneous wall and fluid axial conduction in laminar pipe flow with the assumption that the pipe wall was thin and the velocity profile was the fully developed parabolic profile. They observed that axial conduction inside the tube wall can cause a substantial preheating of both the wall and fluid in the upstream region where there is no external heat input. Conjugate heat transfer when the fluid is turbulent was investigated by Kuznetzov and Belousov [9], Sakakibara and Endoh [10] and Lin and Chow [11].

Recently, more attention has been given to the fluid flow and heat transfer in porous pipes with applications such as transpiration cooling and heat pipes. Usually, the porous pipe wall is of a significant thick-

NOMENCLATURE

C	specific heat
h	heat transfer coefficient
k	thermal conductivity
K	ratio of the thermal conductivities of the wall to the fluid, k_w/k_f
L	length of the porous pipe
Nu_z	local Nusselt number at the solid–fluid interface, $2r_i h_i/k_f$
p	pressure
Pr	Prandtl number, $\mu C/k$
Pe	Peclet number, $Pr\ Re$
q	heat flux
r	radial coordinate
Re	inlet axial Reynolds number, $2r_i w_e/\nu$
Re_i	radial Reynolds number at the solid–fluid interface, $v_i r_i/\nu$
T	temperature
v	radial component of velocity
w	axial component of velocity
z	axial coordinate.

Greek symbols

Δ	dimensionless wall thickness, $(r_o - r_i)/2r_i$
θ	dimensionless temperature
μ	dynamic viscosity
ν	kinematic viscosity
ρ	density.

Subscripts

b	bulk condition
e	entrance condition to the pipe
f	fluid
h	harmonic mean
i	solid–fluid interface
o	outer wall surface
w	wall
z	local condition.

Superscript

*	dimensionless variable.

ness, so that axial wall conduction may have an influence on the heat transfer. In a heat pipe, the thin wick is attached to the inner wall of a thick tube to achieve the capillary force for the liquid return. Blowing and suction occur due to the evaporation and condensation of the working fluid in the heating and cooling segments of the heat pipe. This application was the motivation for the present work. No literature has been found concerning the problem of conjugate heat transfer with blowing and suction at the wall. The objective of the present analysis is to investigate the effect of axial conduction in porous pipes in the combined hydrodynamic and thermal entry lengths with constant heat flux at the outer surface of the pipe wall. In addition, the influence of the Peclet number is stressed due to the inclusion of the axial conduction term in the fluid so as to complete the analysis given by Mori *et al.* [3]. The numerical solution relies on the control volume formulation [12] and an iterative scheme which dealt simultaneously with the fluid and the wall. The numerical results for the Nusselt number and temperature at the interface are presented as in refs. [2, 3]. In addition, the interfacial heat flux for both porous and impermeable wall cases are also given to make the results useful in engineering applications.

MATHEMATICAL FORMULATION

The model used for analysis is shown in Fig. 1. A constant heat flux q_o is applied uniformly along the outer surface of the wall over a finite length L. The fluid enters the inlet of the pipe with a uniform velocity w_e and uniform temperature T_e. The injected and extracted fluid is the same as that of the main pipe flow and is at the interface temperature when it enters or leaves the walls. The equations governing the present problem are the conservation of mass, momentum and energy equations whereby assuming laminar, steady and incompressible flow with constant fluid properties are reduced to the following forms in cylindrical coordinates:

$$\frac{\partial v}{\partial r} + \frac{v}{r} + \frac{\partial w}{\partial z} = 0 \tag{1}$$

$$v\frac{\partial v}{\partial r} + w\frac{\partial v}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial r} + \nu\left[\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial}{\partial r}(rv)\right] + \left\{\frac{\partial^2 v}{\partial z^2}\right\}\right] \tag{2}$$

$$v\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \nu\left[\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial w}{\partial r}\right] + \left\{\frac{\partial^2 w}{\partial z^2}\right\}\right] \tag{3}$$

$$v\frac{\partial T}{\partial r} + w\frac{\partial T}{\partial z} = \frac{k}{\rho C}\left[\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial T}{\partial r}\right] + \left\{\frac{\partial^2 T}{\partial z^2}\right\}\right]. \tag{4}$$

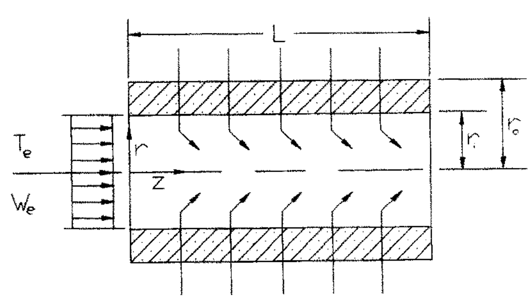

FIG. 1. The schematic model of the heat transfer section of a porous pipe.

It should be noted that in equations (2)–(4), the terms in braces are associated with axial diffusion terms. These terms are neglected when the partially parabolic version is considered but are accounted for in the elliptic case. The thermal conductivity k of the solid is in general different from that of the fluid. At the solid–fluid interface, the harmonic mean is used to determine the thermal conductivity.

The boundary conditions for the case of uniform inlet conditions and constant heat flux at the outer wall are defined as follows:

inlet plane

$$z = 0, \quad 0 < r < r_i$$

$$w = w_e, \quad T = T_e, \quad v = 0 \tag{5}$$

$$r_i < r < r_o, \quad \frac{\partial T_w}{\partial z} = 0; \tag{6}$$

outer wall

$$0 < z < L, \quad r = r_o$$

$$\left.\frac{\partial T}{\partial r}\right|_{\text{wall}} = q_o/k_w = \text{constant}; \tag{7}$$

inner wall

$$0 < z < L, \quad r = r_i$$

$$w = 0 \tag{8}$$

$$v = v_i \begin{cases} v_i > 0 \text{ suction} \\ v_i < 0 \text{ blowing} \\ v_i = 0 \text{ impermeable wall} \end{cases} \tag{9}$$

$$T_f = T_w \tag{10}$$

$$k_f \frac{\partial T_f}{\partial r} = k_w \frac{\partial T_w}{\partial r}; \tag{11}$$

outlet of the tube (elliptic solution): $z = L$

$$r_i < r < r_o, \quad \frac{\partial T_w}{\partial z} = 0 \tag{12}$$

$$0 < r < r_i, \quad \frac{\partial T_f}{\partial z} = 0, \quad p = 0. \tag{13}$$

It should be emphasized that the physical situation corresponding to this problem is the one in which conduction is confined to a finite length of the heated wall, with an adiabatic boundary condition imposed at the upstream and downstream ends of the heated segment. This constraint restricts the results to tubes with short heating sections.

The governing equations (1)–(4) with the appropriate boundary conditions (5)–(13) contain five independent parameters:

Peclet number of the fluid

$$Pe = Pr\, Re = 2r_i w_e \rho_f C_f / k_f;$$

radial Reynolds number at the solid–fluid interface

$$Re_i = v_i r_i / \nu;$$

wall-to-fluid thermal conductivity ratio

$$K = k_w/k_f;$$

thickness of the wall

$$\Delta = (r_o - r_i)/2r_i;$$

heated length

$$L^* = L/2r_i\, Pe.$$

The computations are carried out for Peclet numbers of the fluid of 100 and 1000 with $\Delta = 0.01$ and 0.1 and $K = 1$, 500 and 5000 for different suction and blowing radial Reynolds numbers at the wall. This choice of parameters covers a wide range of possible combinations of fluid and pipe wall properties, flow rates and boundary condition specifications. The negative and positive radial Reynolds numbers at the wall indicate blowing and suction, respectively.

The numerical results can be presented for the following four dimensionless groups in terms of the dimensionless axial distance $z^* = z/2r_i\, Pe$:

interface temperature

$$\theta_i = \frac{T_i - T_e}{q_o r_i / k_f};$$

bulk temperature

$$\theta_b = \frac{T_b - T_e}{q_o r_i / k_f};$$

Nusselt number

$$Nu_z = \frac{2r_i \left(\frac{\partial T}{\partial r}\right)_{r=r_i}}{T_i - T_b}$$

where

$$\left(\frac{\partial T}{\partial r}\right)_{r=r_i} = \frac{q_i}{k_f};$$

heat flux across the interface

$$q_i/q_{\text{mean}}.$$

Although the Nusselt number is traditionally the main dimensionless parameter used in presenting results for conventional convection problems, there is a good justification not to do so for conjugate heat transfer problems. This is due to the fact that q_i, T_b, T_i are all unknowns in the definition of the Nusselt number as given above. Mori *et al.* [3] presented results only for Nu_z and θ_i and therefore their results are of no use for direct engineering applications. From the engineering viewpoint, the most important information is the rate of heat transfer across the interface. So the results are presented in terms of the dimensionless group q_i/q_{mean} as well as the Nusselt number.

The local heat flux across the interface q_i is approximated by

$$\frac{k_h(T_{i,w}-T_{i,f})}{r_i \ln(r_{i,w}/r_{i,f})}$$

with k_h being the harmonic mean of the thermal conductivities of the solid wall and the fluid which is defined as $2k_f \cdot k_w/(k_f+k_w)$ [13]. The radial distance from the pipe axis to the interface is r_i, and $T_{i,w}$ and $T_{i,f}$ are the temperatures at the grid nodes adjacent to the interface on the wall and the fluid sides, respectively. The constant mean heat flux at the interface without considering axial heat conduction in the tube wall q_{mean} is equal to

$$q_{mean} = q_o \frac{r_o}{r_i}.$$

SOLUTION PROCEDURE

The problem is solved as a convection–conduction problem throughout the entire calculation domain, but since the velocities in the solid are zero, the analysis in the solid region was for a pure conduction problem. To account for the discontinuity in the value of the thermal conductivity at the wall–fluid interface, the transport coefficient in the energy equation is evaluated as the harmonic mean of the values on each side of the interface developed by Patankar [13].

The finite difference iteration method of solution developed by Spalding [14] which is employed in the generalized PHOENICS Computational Code [15] is used for the elliptic and partially parabolic solutions of equations (1)–(4) with the boundary specifications given in equations (5)–(13). By this method, the above equations are solved over a square mesh by the finite control volume method outlined in ref. [12]. Finite control volume equations are derived by the integration of the differential equations over an elementary control volume or a cell surrounding a grid node. Upwind differencing is used for the convective terms.

Pressures are solved from a pressure correction equation which yields the pressure change needed to procure velocity changes to satisfy mass continuity. The 'SIMPLEST' practice [14] is employed for the momentum equations. The most significant difference between 'SIMPLEST' and the 'SIMPLE' algorithm [12] is that in the former the control volume formulation coefficients for momentum contain only diffusion contributions, the convection terms being added to the linearized source term of the equations.

The equations are solved by a line-by-line procedure which is similar to Stone's Strongly Implicit Procedure but is free from parameters requiring case-to-case adjustment and is therefore less complex. The temperature is solved in a whole-field manner, and the pressure correction equation is solved in a slab-by-slab manner.

The accuracy of the numerical solution was checked in two ways: the grid spacing was systematically varied and the results for different grid sizes were compared with the extrapolated results of the infinitesimal grid spacing, and the numerical results were compared with a known solution for uniform heat flux and temperature at the interface without axial conduction in the wall or the fluid and with no blowing or suction [16]. The test was passed at a satisfactory level of agreement, within 1% for most of the results.

A number of different uniform grid sizes were chosen to test the accuracy of the solution. After considering the accuracy, computer time and the truncation error, the final grid sizes that were chosen for the presentation of results are as follows:

$20 \times 4 \times 40$ (radial fluid × radial solid × axial) for $\Delta = 0.01$

$20 \times 10 \times 20$ (radial fluid × radial solid × axial) for $\Delta = 0.1$.

A converged solution was obtained by checking the variation of the residuals in terms of the sweep number. When increasing the sweep number to the extent that no further decrease of the residuals is observed, this means the results are converging. For this conjugate problem, 150 sweeps were found to be adequate for convergence of the elliptic solution.

RESULTS AND DISCUSSION

The numerical results were obtained for at least two different values of each of the dimensionless parameters in order to determine the importance of axial conduction. These parameters are: the Peclet number of the fluid, the radial Reynolds number at the solid–fluid interface, the wall-to-fluid thermal conductivity ratio and the dimensionless thickness of the pipe wall. In the present work, only one dimensionless pipe length $L^* = 0.05$ is chosen to avoid the excessive presentation of the results. Axial heat conduction along the pipe wall is usually neglected by practising engineers when designing heat exchangers or other kinds of heat transfer devices because it is very difficult to deal with conjugate heat transfer. Therefore, it is of practical importance to know the errors that may be introduced by this assumption. The numerical results for the three dimensionless variables: the Nusselt number Nu_z, the interface temperature θ_i, and the heat flux across the interface q_i/q_{mean} are given in terms of z^* for different values of K, Δ, Pe and Re_i. The bulk temperature θ_b can be calculated from the information presented.

Figures 2–4 show the effect of the thermal conductivity ratio for $\Delta = 0.1$, $Pe = 100$ and $Re_i = 1.0, 0, -3.0$ on the axial variation of the Nusselt number, the interface temperature and the interfacial heat flux, respectively. The effect of axial conduction becomes more important as the conductivity ratio K increases. As K increases, the results show that the interface temperature distribution changes gradually to a con-

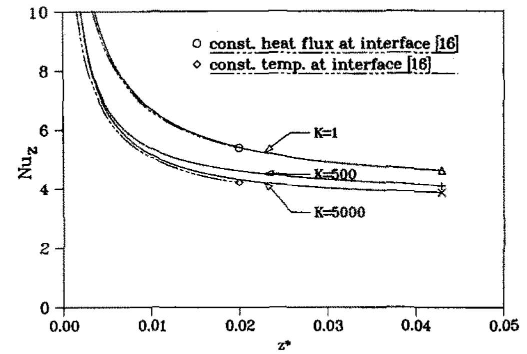

FIG. 2(a). The effect of the thermal conductivity ratio K on the Nusselt number at the interface for $Pe = 100$, $\Delta = 0.1$ and $Re_i = 0$.

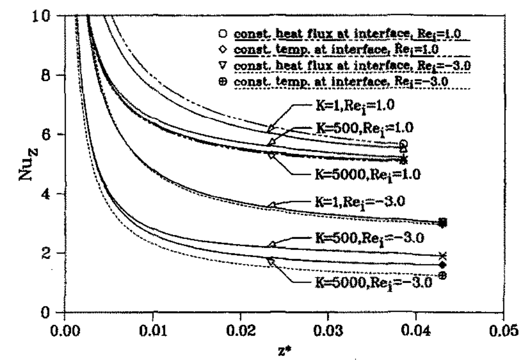

FIG. 2(b). The effect of the thermal conductivity ratio K on the Nusselt number at the interface for $Pe = 100$ and $\Delta = 0.1$ with suction and blowing.

stant temperature along z^* and the Nusselt number approaches that of a constant wall temperature. Physically, when K is very large the axial conduction in the wall has a tendency to make the wall temperature uniform. It is similar to the situation when condensation or evaporation at a wall occurs, which leads to a uniform wall temperature because of the high heat transfer coefficient. The blowing case has a higher interface temperature and the suction case has a lower interfacial temperature in comparison with the case in which the wall is impermeable. At $K = 5000$, the interface temperature distribution becomes constant except for the short entrance region and the Nusselt

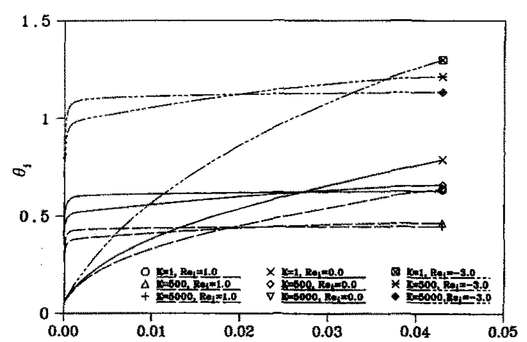

FIG. 3. The effect of the thermal conductivity ratio K on the interface temperature for $Pe = 100$ and $\Delta = 0.1$.

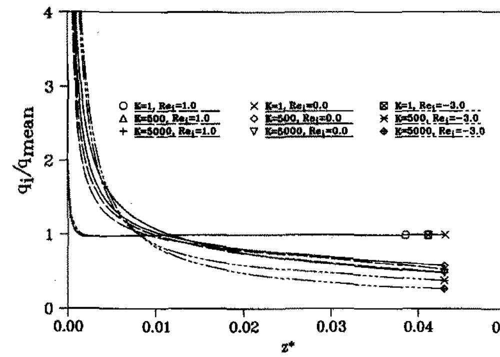

FIG. 4. The effect of the thermal conductivity ratio K on the interfacial heat flux for $Pe = 100$ and $\Delta = 0.1$.

number also approaches the constant temperature results [16]. It is noticed from the present results and those presented in ref. [17] for the case of no axial conduction that blowing at the wall caused a bigger difference between the Nusselt numbers for the cases of constant heat flux and constant wall temperature. So the effect of axial conduction is more pronounced in this case than in other cases. In general, as the thermal conductivity decreases, the Nusselt number increases and the interfacial heat flux tends to become more uniform. For $K = 1$, the results show that the interfacial heat flux almost becomes uniform for all three cases. Also, the Nusselt number approaches the case of constant heat flux at the interface. This is because the axial conduction causes both the interfacial temperature and bulk temperature to increase at approximately the same rate, so that the difference of $T_i - T_b$ and $(\partial T/\partial r)_{r=r_i}$ almost remains unchanged. The numerical results for constant heat flux and temperature at the interface for blowing and suction are computed with the consideration of axial conduction through the fluid. The numerical results for the Nusselt number for $K = 1$ without blowing and suction compare favorably with the numerical results for the case of constant heat flux at the interface as given in ref. [16]. The Nusselt number for $K = 1$ with suction is slightly smaller than the results for constant heat flux at the interface and the results for the case of blowing is slightly greater than the results for constant heat flux at the interface. This is probably because the results for constant heat flux at the interface include the axial heat conduction through the fluid. The results for the Nusselt number with $K \geqslant 1$ are between two limiting cases: constant wall temperature and constant heat flux at the interface. This behavior was first noticed by Mori *et al.* [3] for impermeable walls. The present analysis shows also the same general trend for the cases of blowing and suction. It appears that the influence of axial wall heat conduction must be accounted for if the value of K is large.

Figures 5–7 show the effect of varying the tube wall thickness on the Nusselt number, the interface temperature and the interfacial heat flux, respectively. From these results, it is found that for small Δ the

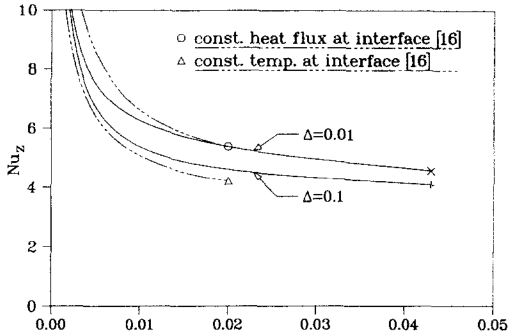

FIG. 5(a). The effect of wall thickness Δ on the Nusselt number at the interface for $Pe = 100$, $K = 500$ and $Re_i = 0$.

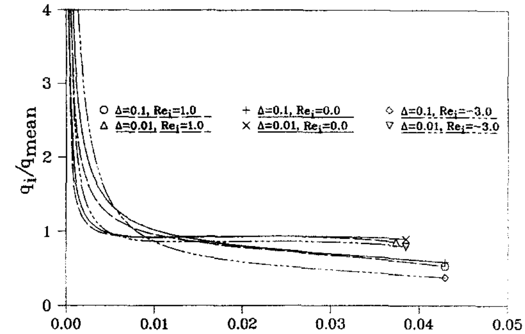

FIG. 7. The effect of the wall thickness on the interfacial heat flux for $Pe = 100$ and $K = 500$.

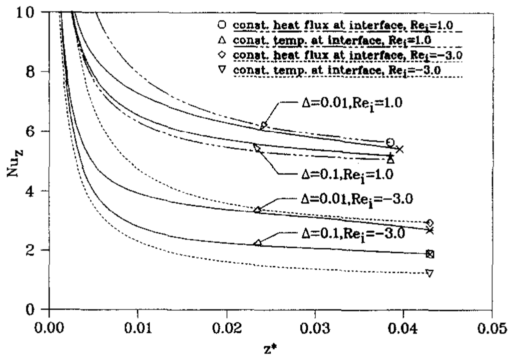

FIG. 5(b). The effect of wall thickness Δ on the Nusselt number at the interface for $Pe = 100$ and $K = 500$ with suction and blowing.

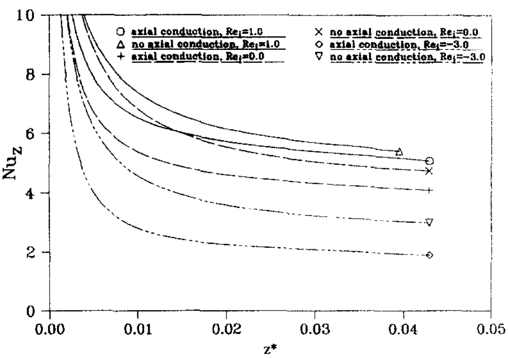

FIG. 8. The effect of axial heat conduction on the Nusselt number at the interface for $Pe = 100$, $K = 500$ and $\Delta = 0.1$.

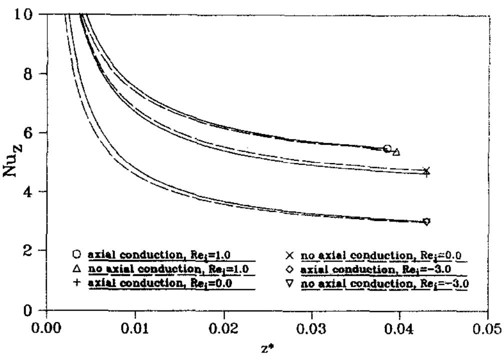

FIG. 9. The effect of axial heat conduction on the Nusselt number at the interface for $Pe = 100$, $K = 1$ and $\Delta = 0.1$.

effect of axial heat conduction through the wall can be neglected due to the fact that the results approach that of the constant heat flux at the interface. Faghri and Sparrow [5] assumed a thin-walled tube, so that all of the results for fully developed flow approach the results for a constant heat flux at the interface. It is found that when the wall thickness increases, the axial wall conduction becomes more important and the interface temperature becomes more uniform because axial heat conduction tends to level out the temperature along the pipe. These results also show that the interfacial heat flux tends to be uniform when the wall becomes thinner. It is interesting to note that all

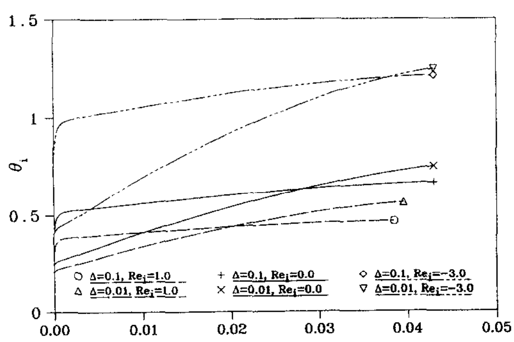

FIG. 6. The effect of the wall thickness Δ on the interface temperature for $Pe = 100$ and $K = 500$.

of the results for different values of Δ are within the two limiting cases: constant heat flux at the interface and constant wall temperature at the interface. It should be mentioned that when $K \to 1$ the axial conduction becomes less important so that the influence of Δ will also become less important. When $K \to \infty$ the influence of Δ will become pronounced. This seems quite reasonable from the physical viewpoint of the problem. It appears from these results that more attention should be given to the blowing case with large K. The trends of the results concerning the influence of the wall thickness agree fairly well with those in ref. [3] for the case of an impermeable wall.

Figures 8–10 show the comparison between the par-

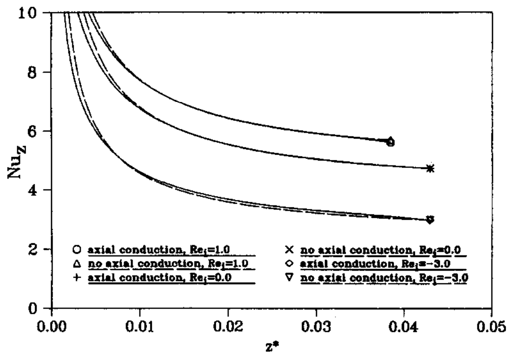

FIG. 10. The effect of axial heat conduction on the Nusselt number at the interface for $Pe = 1000$, $K = 500$ and $\Delta = 0.1$.

tially parabolic and the elliptic solution on the Nusselt number for different values of Pe and K. Figure 8 shows the values of the Nusselt number predicted with and without wall and fluid axial conduction for $Pe = 100$, $\Delta = 0.1$, $K = 500$ and $Re_i = 1.0, 0, -3.0$. It can be seen that when axial conduction is neglected, the Nusselt number is overpredicted along the entire entry region. This effect diminishes as the wall-to-fluid conductivity ratio is reduced as shown in Fig. 9. When the Peclet number is increased to 1000, smaller differences are observed.

CONCLUSIONS

The effects of axial heat conduction have been studied for cases with blowing and suction and with an impermeable wall. It is concluded that large errors will arise in the solution when the axial wall conduction is neglected for blowing cases with large K. As K increases, the results approach the solution of constant wall temperature at the interface without axial wall conduction. For $K = 1$, the local Nusselt number approaches the solution of constant heat flux at the interface. It is confirmed that the effect of axial conduction in the wall can be neglected reasonably when the wall is very thin. But when the wall is very thick, the results approach the solution of constant wall temperature at the interface. Increasing Pe by a factor of 10 reduces the effect of axial conduction much more than a corresponding reduction of Δ or K.

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CONDUCTION AXIALE SIMULTANEE DANS LE FLUIDE ET DANS LA PAROI DU TUBE POUR UN ECOULEMENT CONVECTIF LAMINAIRE AVEC SOUFFLAGE OU ASPIRATION A LA PAROI

Résumé—On donne des solutions numériques pour le transfert de chaleur conjugué dans un tuyau perméable ayant un écoulement interne laminaire avec soufflage ou aspiration à la surface interne du tube et un flux thermique constant à la surface externe. L'effet de la conduction axiale simultanée dans la paroi et dans le fluide a été étudié pour les longueurs d'entrée hydrodynamique et thermique. Les résultats montrent que le rapport des conductivités thermiques du tube et du fluide, et l'épaisseur de la paroi peuvent devenir des paramètres significatifs dans le transfert thermique quand le nombre de Péclet est petit, spécialement dans le cas du fluide injecté dans le tube. On constate aussi que l'effet de la conduction axiale dans la paroi, pour le cas du flux thermique constant à la surface externe, peut être négligé quand l'épaisseur de la paroi est faible et quand le rapport de la conductivité de la paroi à celle du fluide approche l'unité.

GLEICHZEITIGE AXIALE WÄRMELEITUNG IM FLUID UND IN DER ROHRWAND BEI LAMINARER ERZWUNGENER KONVEKTION MIT EINBLASUNG UND ABSAUGUNG AN DER WAND

Zusammenfassung—Für den konjugierten Wärmeübergang in einem porösen Rohr bei laminarer Strömung mit Einblasung oder Absaugung an der Rohrinnenwand und einer konstanten Wärmestromdichte an der Außenwand werden numerische Lösungen dargestellt. Der Einfluß der gleichzeitigen axialen Wärmeleitung in der Wand und im Fluid wurde für kombinierte hydrodynamische und thermische Einlauflängen untersucht. Die Ergebnisse zeigen, daß das Verhältnis der Wärmeleitfähigkeiten von Rohrwand und Fluid sowie die Wanddicke einen bedeutenden Einfluß auf den Wärmeübergang haben können, wenn die Peclet-Zahl klein ist, insbesondere wenn Fluid in das Rohr injiziert wird. Weiterhin wird gezeigt, daß der Einfluß der axialen Wärmeleitung für den Fall einer konstanten Wärmestromdichte an der Außenwand vernachlässigt werden kann, wenn die Wanddicke klein ist und die Wärmeleitfähigkeiten von Wand und Fluid etwa gleich sind.

СОВМЕСТНАЯ АКСИАЛЬНАЯ ТЕПЛОПРОВОДНОСТЬ В ЖИДКОСТИ И СТЕНКЕ ТРУБЫ ПРИ ВЫНУЖДЕННОКОНВЕКТИВНОМ ЛАМИНАРНОМ ТЕЧЕНИИ СО ВДУВОМ И ОТСОСОМ У СТЕНКИ

Аннотация—Приведены численные решения задачи сопряженного теплопереноса в пористой трубе при ламинарном течении со вдувом или отсосом у внутренней поверхности и с постоянным тепловым потоком на внешней поверхности. Изучено влияние совместной аксиальной теплопроводности через стенку и жидкость на совокупность длины гидродинамического и теплового начальных участков. Результаты показывают, что отношение теплопроводности стенки трубы к теплопроводности жидкости и толщина трубы могут в значительной мере определять теплоперенос при низких числах Пекле, особенно пви вдуве жидкости в трубу. Показано также, что воздействием аксиальной теплопроводности в стенке можно пренебречь в случаях постоянного теплового потока на ее внешней поверхности при малой толщине стенки и при отношении теплопроводностей, близком к единице.